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Derivation of the Adomian decomposition method using the homotopy analysis method

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Abstract

Adomian decomposition method has been used intensively to solve nonlinear boundary and initial value problems. It has been proved to be very efficient in generating series solutions of the problem under consideration under the assumption that such series solution exits. However, very little has been done to address the mathematical foundation of the method and its error analysis.

In this article the mathematical derivation of the method using the homotopy analysis method is presented. In addition, an error analysis is addressed as well as the convergence criteria.

Keywords: Adomian decomposition method; Homotopy analysis method; Error analysis; Convergence criteria

1. Introduction

Nonlinear differential equations are usually arising from mathematical modeling of many frontier physical systems. In most cases, analytic solutions of these differential equations are very difficult to achieve. Common analytic procedures linearize the system or assume the nonlinearities are relatively insignificant. Such procedures change the actual problem to make it tractable by the conventional methods. This changes, some times seriously, the solution. Generally, the numerical methods such as Rung–Kutta method are based on discretization techniques, and they only permit us to calculate the approximate solutions for some values of time and space variables, which causes us to overlook some important phenomena such as chaos and bifurcation, in addition to the intensive computer time required to solve the problem. The above drawbacks of linearization and numerical methods arise the need to search for an alternative techniques to solve the nonlinear differential equations, namely, the analytic solution methods, such as the perturbation method, the iteration variational method [11,13–15] and the Adomian decomposition method.

The Adomian decomposition method [1–3] is quantitative rather than qualitative, analytic, requiring neither linearization nor perturbation and continuous with no resort to discretization. It consists of splitting the given equation into linear and nonlinear parts, inverting the highest-order derivative operator contained

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in the linear operator on both sides, identifying the initial and/or boundary conditions and the terms involving the independent variables alone as initial approximation, decomposing the unknown function into a series whose components are to be determined, decomposing the nonlinear function in terms of special polynomials called Adomian's polynomials, and finding the successive terms of the series solution by recurrent relation using Adomian polynomials.

The method has been used to derive analytical solution for nonlinear ordinary differential equations [8,20,21,30] as well as partial differential equations [4,7,22,23,31]. A modified version of the method was used to derive the analytic solution for partial and ordinary differential equations [34,38–40]. Application of the method to fractional differential equations was first introduced by Shawagfeh [28,29] .Other applications of the method in various fields of applied sciences can be found in [24,25,27,32,33,35–37].

Error analysis and convergence criteria of the method was investigated by several authors. In [9,10] Cherrualt investigate the convergence of the method when applied to a special class of boundary value problems. The convergence of Adomian's decomposition method, as applied to the special problem of periodic temperature fields in heat conductors, was investigated in [26]. However, in [12] it was shown that ADM does not converge in general, in particular, when the method is applied to linear operator equations. It was also shown that Adomian's decomposition method is equivalent to Picard iteration method, and therefore it might diverge.

The Homotopy Analysis Method (HAM), which was first introduced by Liao (see [16–19] and the references therein), is another technique used to derive an analytic solution for nonlinear operators. It consists of introducing embedding operators and embedding parameters where the solution is assumed to depend continuously on these parameters. The method has been used intensively by many authors and proved to be very effective in deriving an analytic solution of nonlinear differential equations [8,16,17].

However, although Adomian decomposition method (ADM) has been used intensively to solve nonlinear problems, very little is known about the theory behind this method and it's convergence. This article is an attempt to give a mathematical derivation of the ADM method using Homotopy analysis method (HAM). In the next section we will give a brief description of the ADM, while in Section 3 we will give a systematic description of the Homotopy analysis method (HAM). While in Section 4, we will give the main result of this article which is the derivation of the ADM using HAM. Conclusion remarks are presented in Section 5.

2. Adomian decomposition method (ADM)

Adomian decomposition method (ADM) depends on decomposing the nonlinear differential equation

$$F(x, y(x)) = 0 \tag{1}$$

into the two components

$$L(y(x)) + N(y(x)) = 0, \tag{2}$$

where L and N are the linear and the nonlinear parts of F respectively. The operator L is assumed to be an invertible operator. Solving for $L(y)$ leads to

$$L(y) = -N(y). \tag{3}$$

Applying the inverse operator L^{-1} on both sides of Eq. (3) yields

$$y = -L^{-1}(N(y)) + \varphi(x), \tag{4}$$

where $\varphi(x)$ is the constant of integration satisfies the condition $L(\varphi) = 0$. Now assuming that the solution y can be represented as infinite series of the form

$$y = \sum_{n=0}^{\infty} y_n. \tag{5}$$

Furthermore, suppose that the nonlinear term $N(y)$ can be written as infinite series in terms of the Adomian polynomials A_n of the form

$$N(y) = \sum_{n=0}^{\infty} A_n, \tag{6}$$

where the Adomian polynomials A_n of $N(y)$ are evaluated using the formula [1]:

$$A_n(x) = \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}\lambda^n} N\left(\sum_{n=0}^{\infty}(\lambda^n y_n)\right)\Bigg|_{\lambda=0}.$$

Then substituting Eqs. (5) and (6) in Eq. (4) gives

$$\sum_{n=0}^{\infty} y_n = \varphi(x) - L^{-1}\left(\sum_{n=0}^{\infty} A_n\right). \tag{7}$$

Then equating the terms in the linear system of Eq. (7) gives the recurrent relation

$$y_0 = \varphi(x)y_{n+1} = -L^{-1}(A_n) \quad n \geqslant 09. \tag{8}$$

However, in practice all the terms of series (7) cannot be determined, and the solution is approximated by the truncated series $\sum_{n=0}^{N} y_n$. This method has been proven to be very efficient in solving various types of nonlinear boundary and initial value problems, see for examples [4–7].

3. Homotopy analysis method

In what follows, a description of the Homotopy analysis method as it appears in various literatures [16–19] will be presented. Consider a nonlinear differential operator $\widetilde{N}$, let $\hbar \neq 0$ and λ be complex numbers, and $A\lambda$ and $B\lambda$ be two complex functions analytic in the region $|\lambda| \leqslant 1$, satisfying

$$A(0) = B(0) = 0, \quad A(1) = B(1) = 1, \tag{9}$$

respectively. Besides, let

$$A(\lambda) = \sum_{k=1}^{\infty} \alpha_{1,k}\lambda^k, \quad B(\lambda) = \sum_{k=1}^{\infty} \beta_{1,k}\lambda^k \tag{10}$$

denote the Maclaurin's series of $A\lambda$ and $B\lambda$ respectively. Because $A\lambda$ and $B\lambda$ are analytic in the region $|\lambda| \leqslant 1$, therefore we have

$$A(1) = \sum_{k=1}^{\infty} \alpha_{1,k} = 1, \quad B(1) = \sum_{k=1}^{\infty} \beta_{1,k} = 1. \tag{11}$$

The above defined complex functions $A\lambda$ and $B\lambda$ are called the embedding functions and λ is the embedding parameter.Consider the nonlinear differential equation in general form

$$\widetilde{N}(y(x)) = 0, \quad x \in \Omega, \tag{12}$$

where $\widetilde{N}$ is a differential operator, $y(x)$ is a solution defined in the region $x \in \Omega$. To solve Eq. (12), using the homotopy analysis method we construct the equation

$$[1 - B(\lambda)]\{\pounds[\tilde{y}(x,\lambda) - y_0(x)]\} = \hbar A(\lambda)\widetilde{N}[\tilde{y}(x,\lambda)], \tag{13}$$

where £ is a properly selected auxiliary linear operator satisfying

$$\pounds(0) = 0 \tag{14}$$

and $\hbar \neq 0$ is an auxiliary parameter, y_0(x) is an initial approximation. Using the facts $A(0) = 0$ and $B(0) = 0$, Eq. (13) gives

$$\pounds[\tilde{y}(x,0) - y_0(x)] = 0,$$

or equivalently

$$\tilde{y}(x,0) = y_0(x). \tag{15}$$

Similarly, when $\lambda = 1$, Eq. (13) is the same as Eq. (12) so that we have

$$\tilde{y}(x,1) = y(x). \tag{16}$$

Suppose that Eq. (12) has solution $\tilde{y}(x,\lambda)$ that converges for all $0 \leqslant \lambda \leqslant 1$ for properly selected $\hbar$, $A\lambda$ and $B\lambda$. Suppose further that $\tilde{y}(x,\lambda)$ is infinitely differentiable at $\lambda = 0$, that is

$$\tilde{y}_0^k(x) = \left.\frac{\partial^k \tilde{y}(x,\lambda)}{\partial \lambda^k}\right|_{\lambda=0}, \quad k = 0, 1, 2, 3, \ldots, \tag{17}$$

exists for all $k = 0, 1, 2, \ldots$ Thus as λ increases from 0 to 1, the solution $\tilde{y}(x,\lambda)$ of Eq. (13) varies continuously from the initial approximation $y_0(x)$ to the solution $y(x)$ of the original Eq. (12). Clearly, Eq. (15) and Eq. (16) gives an indirect relation between the initial approximation $y_0(x)$ and the general solution $y(x)$. The homotopy analysis method depends on finding a direct relationship between the two solutions which can be described as follows.

Consider the Maclaurin's series of $\tilde{y}(x,\lambda)$ about $\lambda = 0$

$$\tilde{y}(x,\lambda) = \tilde{y}(x,0) + \sum_{k=1}^{\infty} \left(\left.\frac{\partial^k \tilde{y}(x,\lambda)}{\partial \lambda^k}\right|_{\lambda=0} \right) \frac{\lambda^k}{k!}. \tag{18}$$

Assuming that the series above converges at $\lambda = 1$, we have by Eqs. (15) and (16) the relationship

$$y(x) = y_0(x) + \sum_{m=1}^{\infty} y_m(x), \tag{19}$$

where

$$y_m(x) = \frac{\tilde{y}_0^m(x)}{m!} = \frac{\left.\frac{\partial^m \tilde{y}(x,\lambda)}{\partial \lambda^m}\right|_{\lambda=0}}{m!}, \quad m \geqslant 1. \tag{20}$$

To derive the governing equation of $y_m(\mathrm{x})$, we differentiate Eq. (12) m times with respect to λ we get

$$\sum_{k=0}^{m} \binom{m}{k} \frac{\mathrm{d}^k[1 - B(\lambda)]}{\mathrm{d}\lambda^k} \frac{\mathrm{d}^{m-k}}{\mathrm{d}\lambda^{m-k}} \{£[\tilde{y}(x,\lambda)] - £[y_0(x)]\} = \hbar \sum_{k=0}^{m} \binom{m}{k} \frac{\mathrm{d}^k A(\lambda)}{\mathrm{d}\lambda^k} \frac{\mathrm{d}^{m-k} \tilde{N}[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^{m-k}}. \tag{21}$$

Further dividing Eq. (21) by $m!$ and then setting $\lambda = 0$, we have the so called mth-order deformation equations

$$£\left[y_m(x) - \sum_{k=1}^{m-1} \beta_{1,k} y_{m-k}(x) \right] = R_m(x), \tag{22}$$

where $R_m(x)$ in fact depends on the previous calculated values of $y_0(x), y_1(x), \ldots, y_{m-1}(x)$ and given by

$$R_m(x) = \hbar \sum_{k=1}^{m} \alpha_{1,k} h_{m-k}(x) \tag{23}$$

and $h_k(x)$ are the Homotopy polynomials and given by

$$h_k(x) = \frac{1}{k!} \left.\frac{\mathrm{d}^k \tilde{N}[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^k}\right|_{\lambda=0}. \tag{24}$$

It is very important to emphasize that Eq. (22) is linear. If the first $(m-1)$th-order approximations have been obtained, the left hand side $R_m(x)$ will be obtained. So, using the selected initial approximation $y_0(x)$, we can obtain $y_1(x), y_2(x), y_3(x), \ldots$, one after the other in order. Therefore, according to Eq. (22), we, in fact, convert the original nonlinear problem given by Eq. (12) into an infinite sequence of linear sub-problems governed by Eq. (22). We emphasize that HAM provides us with great freedom and large flexibility to select the non-zero auxiliary parameters $\hbar$, the embedding functions $A\lambda$ and $B\lambda$, the initial approximation $y_0(x)$ and the auxiliary linear operators £.

4. The mathematical derivation of the Adomian decomposition method

The Adomian decomposition method can be derived using the Homotopy analysis method using the following theorem:

Theorem 1. *Let the embedding operators $A\lambda$ and $B\lambda$ be given by*

$$A(\lambda) = \lambda \quad \text{and} \quad B(\lambda) = \lambda \tag{25}$$

and let the auxiliary parameter $\hbar = -1$, then

$$A_n(x) = h_n(x), \tag{26}$$

i.e., the HAM polynomials will be reduced to the Adomian polynomials.

Proof. Assume that the solution of Eq. (1) depends continuously on the parameter λ for $0 \leqslant \lambda \leqslant 1$, assume further that the solution is given by $\omega(x, \lambda)$ and this solution is analytic at $\lambda = 0$ so that

$$\omega(x, \lambda) = \sum_{k=0}^{\infty} \frac{\lambda^k}{k!} \frac{\partial^k \omega(x, \lambda)}{\partial \lambda^k}\bigg|_{\lambda=0}.$$

With the above choice of the embedding parameters, Eq. (13) will be

$$[1 - \lambda]\{L[\omega(x, \lambda)] - L[(y_0)]\} = -\lambda F[\omega(x, \lambda)], \tag{27}$$

where the initial guess y_0 is the solution of the linear operator

$$L[\omega(x, 0)] = 0$$

it is clear that when $\lambda = 0$, the solution of Eq. (27) will be y_0 and when $\lambda = 1$, the solution will be $y(x)$ which is the solution of the original nonlinear equation $F[\omega(x, \lambda)] = 0$. Differentiating both sides of Eq. (27) with respect to λ will lead to the following relation for the HAM polynomials:

$$[1 - \lambda]\frac{\partial L[\omega(x, \lambda)]}{\partial \lambda} - \{L[\omega(x, \lambda) - L(y_0)]\} = -F[\omega(x, \lambda)] - \lambda \frac{\partial F[\omega(x, \lambda)]}{\partial \lambda}$$

and when $\lambda = 0$ we will have the equation for y_1 which is

$$L\left[\frac{\partial \omega(x, \lambda)}{\partial \lambda}\bigg|_{\lambda=0}\right] = L[y_1] = -F[y_0] = -A_0, \tag{28}$$

or

$$y_1(x) = -L^{-1}(A_0).$$

To derive the nth equation for the nth term of the HAM polynomial, we differentiate Eq. (27) with respect to λ n-times to obtain

$$\sum_{k=0}^{n} \binom{n}{k} \frac{\partial^k (1 - \lambda)}{\partial \lambda^k} \frac{\partial^{n-k}}{\partial \lambda^{n-k}} \{L[\omega(x, \lambda)] - L[y_0] = -1 \sum_{k=0}^{n} \binom{n}{k} \frac{\partial^k (\lambda)}{\partial \lambda^k} \frac{\partial^{n-k} F[\omega(x, \lambda)]}{\partial \lambda^{n-k}}. \tag{29}$$

Simplifying last equation, Eq. (29) and divide by $m!$ then set $\lambda = 0$, we obtain the following linear equation for y_n:

$$L(y_n) = -A_{n-1}, \tag{30}$$

or

$$y_n = -L^{-1}(A_{n-1}), \tag{31}$$

where

$$A_n = \frac{1}{n!} \frac{\partial^n F[\omega(x, \lambda)]}{\partial \lambda^n}\bigg|_{\lambda=0} = \delta_n, \tag{32}$$

which completes the proof. □

Theorem 1 above gives a great freedom to choose the linear operator £ when the Adomian decomposition method is to be applied. Application of this theorem to a certain class of differential equations, namely, differential equations with infinite domain can be found in [6].

In this section the question of convergence of the Adomian decomposition method will be addressed.

Theorem 2. *If the series*

$$y_0(x) + \sum_{m=1}^{\infty} y_m(x) \tag{33}$$

is convergent, it must be a solution of Eq. (1).

Proof. By Eq. (22) we have

$$\begin{aligned}\sum_{m=1}^{\infty} R_m(x) &= \sum_{m=1}^{\infty} £\left[y_m(x) - \sum_{k=1}^{m-1} \beta_{1,k} y_{m-k}(x)\right] = £\left[\sum_{m=1}^{\infty} y_m(x) - \sum_{m=1}^{\infty}\sum_{k=1}^{m-1} \beta_{1,k} y_{m-k}(x)\right] \\ &= £\left[\sum_{m=1}^{\infty} y_m(x) - \sum_{k=1}^{\infty}\sum_{m=k+1}^{m-1} \beta_{1,k} y_{m-k}(x)\right] = £\left[\sum_{m=1}^{\infty} y_m(x) - \sum_{k=1}^{\infty} \beta_{1,k} \sum_{m=1}^{m-1} y_m(x)\right] \\ &= £\left[\left(1 - \sum_{k=1}^{\infty} \beta_{1,k}\right) \sum_{m=1}^{\infty} y_m(x)\right].\end{aligned} \tag{34}$$

Recall that

$$\beta(\lambda) = \lambda.$$

Therefore,

$$\beta_{1,k} = \begin{cases} 0, & \text{when } k = 0, \\ 1, & \text{when } k = 1, \\ 0, & \text{when } k > 1, \end{cases}$$

which gives by Eq. (34)

$$\sum_{m=1}^{\infty} R_m(x) = 0. \tag{35}$$

On the other hand, we have by Eqs. (23) and (24) that

$$\begin{aligned}\sum_{m=1}^{\infty} R_m(x) &= \sum_{m=1}^{\infty} \hbar \sum_{k=1}^{m} \alpha_{1,k} h_{m-k}(x) = \hbar \sum_{m=1}^{\infty} \alpha_{1,k} \sum_{m=k}^{\infty} h_{m-k}(x) = \hbar \sum_{m=1}^{\infty} \alpha_{1,k} \sum_{m=0}^{\infty} h_m(x) \\ &= \hbar \sum_{m=1}^{\infty} \alpha_{1,k} \sum_{m=k}^{\infty} \frac{1}{m!} \left.\frac{\mathrm{d}^m \tilde{N}[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^m}\right|_{\lambda=0}.\end{aligned} \tag{36}$$

Again, recall that

$$A(\lambda) = \lambda.$$

Therefore,

$$\alpha_{1,k} = \begin{cases} 0, & \text{when } k = 0, \\ 1, & \text{when } k = 1, \\ 0, & \text{when } k > 1. \end{cases}$$

Therefore, $\sum_{k=1}^{\infty} \alpha_{1,k} = 1$, Thus the above expression becomes

$$\sum_{m=1}^{\infty} R_m(x) = \hbar \sum_{m=1}^{\infty} h_m(x) = \hbar \sum_{m=0}^{\infty} \frac{1}{m!} \left.\frac{\mathrm{d}^m F[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^m}\right|_{\lambda=0}. \tag{37}$$

Note that $\hbar = -1$, Therefore, By Eqs. (35) and (36) we have

$$\sum_{m=0}^{\infty} \frac{1}{m!} \frac{\mathrm{d}^m F[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^m}\bigg|_{\lambda=0} = 0. \tag{38}$$

In addition, $\tilde{y}(x,\lambda)$ is not a solution of Eq. (1) in general when $\lambda \neq 1$. Now define $\Delta(x,\lambda) = F[\tilde{y}(x,\lambda)] - F[y(x)] = F[\tilde{y}(x,\lambda)]$, as a residual error of Eq. (1) then the Maclaurin's series of this residual about $\lambda = 0$ is

$$\sum_{m=0}^{\infty} \frac{\mathrm{d}^m \Delta(x,\lambda)}{\mathrm{d}\lambda^m}\bigg|_{\lambda=0} \frac{\lambda^m}{m!} = \sum_{m=0}^{\infty} \frac{\mathrm{d}^m F[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^m}\bigg|_{\lambda=0} \frac{\lambda^m}{m!}. \tag{39}$$

According to Eq. (33), the above Maclaurin's series converges at $\lambda = 1$, say

$$\Delta(x,1) = F[\tilde{y}(x,1)] = \sum_{m=0}^{\infty} \frac{1}{m!} \frac{\mathrm{d}^m F[\tilde{y}(x,\lambda)]}{\mathrm{d}\lambda^m}\bigg|_{\lambda=0} = 0, \tag{40}$$

which means that

$$y(x) = \tilde{y}(x,1) = y_0(x) + \sum_{m=1}^{\infty} y_m(x)$$

must be a solution of Eq. (1), which completes the proof. □

To estimate whether the series converges or diverges, one can use the following theorem:

Theorem 3. *If the series $y_0(x) + \sum_{m=1}^{\infty} y_m(x)$ converges then the following two sequences*:

$$\varkappa_k = \sum_{m=1}^{k} R_m(x),$$
$$v_k = \sum_{m=1}^{k} h_m(x)$$

where $R_m(x)$ and $h_m(x)$ are defined by Eqs. (23) and (24) converge to zero.

Proof. The proof of this theorem is a subsequent of Eqs. (35) and (37). □

The above analysis shows that one has a variety of choices for the linear operator L and therefore a variety of choices for the initial estimation $y_0(x)$ to start the Adomian decomposition iteration process. For example, consider the non linear problem describes the fluid flow over a flat plate, know as the Balsius problem which is given by

$$f'''(\eta) + \frac{1}{2} f(\eta) f''(\eta) = 0,$$
$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1.$$

This problem was investigated in [6,16,34]. When the classical decomposition method is used, the linear operator is given by

$$L(f) = f'''$$

leads to a polynomial that approximate the solution in a finite domain only and the solution diverges as $t \to \infty$. When a different linear operator was chosen such that the boundary condition at infinity is taken into consideration, the operator is given by

$$L(f) = f''' + \beta f''$$

the initial solution $f_0(\eta)$ is given by

$$f_0(\eta) = \frac{\mathrm{e}^{-\beta\eta}}{\beta} + \eta - \frac{1}{\beta},$$

where the parameter β was chosen to improve the rate of convergence of the iteration process and it was chosen to be $\beta = 3$. It is clear that the function satisfies the initial conditions $f_0(0) = 0$, $f_0'(0) = 0$ and the boundary condition at infinity $f_0'(\infty) = 1$. The sequence of functions $f_n(\eta)$ for $n \geqslant 1$ obtained by the recurrence relation of Adomian decomposition method satisfies the initial conditions $f_n(0) = 0$, $f_n'(0) = 0$ and the boundary condition at infinity $f_n'(\infty) = 0$. It was shown that the solution $f(\eta) = \sum_{n=0}^{N} f_n(\eta)$ converges for all values of $0 \leqslant \eta < \infty$. Full details of the method can be found in [6].

5. Conclusion

In this article, the homotopy analysis method was used to derive the Adomian decomposition method. In fact it was shown that the Adomian decomposition method is a special case of the homotopy analysis method. In addition the criteria for the convergence of the homotopy analysis method was shown to be the same criteria for the convergence of the Adomian decomposition method. The above analysis also shows that one has a variety of choices for the linear operator L and therefore a variety of choices for the initial estimation $y_0(x)$ to start the Adomian decomposition iteration process.

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References

[1] G. Adomian, Solving Frontier Problems on Physics: The Decomposition Method, Kluwer Academic Publisher, Boston, 1994.
[2] G. Adomian, A review of the decomposition method in applied mathematics, J. Math. Anal. Appl. 135 (1988) 501–544.
[3] G. Adomian, Solution of the Thomas–Fermi equation, Appl. Math. Lett. 11 (3) (1998) 31–133.
[4] K. Al Khaled, F. Allan, Construction of solution of shallow water equation by decomposition method, Math. Comput. Sim. 66 (6) (2004) 479–486.
[5] K. AL Khaled, F. Allan, Decomposition method for solving nonlinear integro-differential equations, Kor. J. Appl. Math. Comput. 19 (1) (2005) 415–425.
[6] F. Allan, On the analytic solution of non-linear boundary value problem with infinite domain, in: Proceedings of the 6th Annual Conference of UAEU, 9 (April) 2005.
[7] F. Allan, K. Al Khaled, On the analytic solution of fully developed shock wave equation, J. Comput. Appl. Math. 192 (2) (2006) 301–309.
[8] F. Allan, M. Syam, On the analytic solutions of the nonhomogeneous Blasius problem, J. Comput. Appl. Math. 182 (2) (2005) 362–371.
[9] Y. Cherrualt, Convergence of Adomian's method, Kybernetes 18 (1989) 31–38.
[10] Y. Cherrualt, G. Adomian, Decomposition methods: a new proof of convergence, Math. Comput. Model. 18 (1993) 103–106.
[11] J.H. He, Variational iteration method: a kind of nonlinear analytical technique: some examples, Int. Non-Linear Mech. 344 (1999) 699–708.
[12] M.A. Golberg, A note on the decomposition method for operator equations, Appl. Math. Comput. 106 (2–3) (1999) 215–220.
[13] J.H. He, Variational iteration method for autonomous ordinary differential system, Appl. Math. Comput. 114 (2) (2000) 115–123.
[14] J.H. He, Commun. Nonlinear Sci. Num. Simul. 3 (2002) 107–120.
[15] J.H. He, Homotopy perturbation method: a new nonlinear technique, Appl. Math. Comput. 135 (2003) 73–79.
[16] S.J. Liao, An explicit, totally analytic approximate solution for Blasius viscous flow problems, Int. J. Non-Linear Mech. 34 (1999) 759–778.
[17] S.J. Liao, K.F. Chung, Analytic solution for nonlinear progressive waves in deep water, J. Eng. Math. 45 (2) (2003).
[18] S.J. Liao, An explicit analytic solution to the Thomas–Fermi equation, Appl. Math. Comput. 144 (2003) 433–444.
[19] S.J. Liao, A. Compo, Analytic solution of the temperature distribution in Blasuis viscous flow problem, J. Fluid Mech. 453 (2002) 411–425.
[20] D. Lensic, The decomposition method for Cauchy reaction–diffusion problems, Appl. Math. Lett. 20 (4) (2007) 412–418.
[21] D. Lensic, The decomposition method for initial value problems, Appl. Math. Com. 181 (11) (2006) 206–213.
[22] D. Lensic, Decomposition methods for non-linear, non-characteristic Cauchy heat problems, Commun. Nonlinear Sci. Num. Sim. 10 (6) (2005) 581–596.
[23] D. Lensic, The decomposition method for Cauchy advection–diffusion problems, Comput. Math. Appl. 49 (4) (2005) 525–537.
[24] D. Lensic, P.J. Heggs, A Decomposition method for power-law fin-type problems, Int. Commun. Heat Mass Transfer 31 (5) (2004) 673–682.

[25] D. Lensic, The decomposition method for forward and backward time-dependent problems, J. Comput. Appl. Math. 147 (1) (2002).
[26] D. Lensic, Convergence of Adomian's decomposition method: periodic temperatures, Comput. Math. Appl. 44 (1–2) (2002) 13–24.
[27] D. Lensic, L. Elliott, The decomposition approach to inverse heat conduction, J. Math. Anal. Appl. 232 (1) (1999) 82–98.
[28] S. Momani, N. Shawagfeh, Decomposition method for solving fractional Riccati differential equations, Appl. Math. Comput. 182 (2) (2006) 1083–1092.
[29] Nabil T. Shawagfeh́, Analytical approximate solutions for nonlinear fractional differential equations, Appl. Math. Comput. 131 (2–3) (2002) 517–529.
[30] N.T. Shawagfeh, Analytic approximate solution for a nonlinear oscillator equation, Comput. Math. Appl. 31 (6) (1996) 135–141.
[31] A.M. Wazwaz, Padé approximants and Adomian decomposition method for solving the Flierl–Petviashivili equation and its variants, Appl. Math. Comput. 182 (2) (2006) 1812–1818.
[32] A.M. Wazwaz, A comparison study between the modified decomposition method and the traditional methods for solving nonlinear integral equations, Appl. Math. Comput. 181 (2) (2006) 1703–1712.
[33] A.M.Wazwaz, A comparison between the variational iteration method and Adomian decomposition method, J. Comput. Appl., in press, doi:10.1016/j.cam.2006.07.018.
[34] A.M. Wazwaz, The modified decomposition method and Padé approximants for a boundary layer equation in unbounded domain, Appl. Math. Comput. 177 (2) (2006) 737–744.
[35] A.M. Wazwaz, The modified decomposition method for analytic treatment of differential equations, Appl. Math. Comput. 173 (1) (2006) 165–176.
[36] A.M. Wazwaz, Adomian decomposition method for a reliable treatment of the Bratu-type equations, Appl. Math. Comput. 166 (3) (2005) 652–663.
[37] A.M. Wazwaz, Adomian decomposition method for a reliable treatment of the Emden–Fowler equation, Appl. Math. Comput. 161 (2) (2005) 543–560.
[38] A.M. Wazwaz, Salah M. El-Sayed, A new modification of the Adomian decomposition method for linear and nonlinear operators, Appl. Math. Comput. 122 (3) (2001) 393–405.
[39] A.M. Wazwaz, The modified decomposition method applied to unsteady flow of gas through a porous medium, Appl. Math. Comput. 118 (2–3) (2001) 123–132.
[40] A.M. Wazwaz, A reliable modification of Adomian decomposition method, Appl. Math. Comput. 102 (1) (1999) 77–86.