Preface

It is well-known that perturbation and asymptotic approximations of nonlinear problems often break down as nonlinearity becomes strong. Therefore, they are only valid for weakly nonlinear ordinary differential equations (ODEs) and partial differential equations (PDEs) in general.

The homotopy analysis method (HAM) is an analytic approximation method for highly nonlinear problems, proposed by the author in 1992. Unlike perturbation techniques, the HAM is independent of any small/large physical parameters at all: one can always transfer a nonlinear problem into an infinite number of linear sub-problems by means of the HAM. Secondly, different from all of other analytic techniques, the HAM provides us a convenient way to guarantee the convergence of solution series so that it is valid even if nonlinearity becomes rather strong. Besides, based on the homotopy in topology, it provides us extremely large freedom to choose equation type of linear sub-problems, base function of solution, initial guess and so on, so that complicated nonlinear ODEs and PDEs can often be solved in a simple way. Finally, the HAM logically contains some traditional methods such as Lyapunov's small artificial method, Adomian decomposition method, the δ-expansion method, and even the Euler transform, so that it has the great generality. Therefore, the HAM provides us a useful tool to solve highly nonlinear problems in science, finance and engineering.

This book consists of three parts. In Part I, the basic ideas of the HAM, especially its theoretical modifications and developments, are described, including the optimal HAM approaches, the theorems about the so-called homotopy-derivative operator and the different types of deformation equations, the methods to control and accelerate convergence, the relationship to Euler transform, and so on.

In Part II, inspirited by so many successful applications of the HAM in different fields and also by the ability of "computing with functions instead of numbers" of computer algebra system like Mathematica and Maple, a Mathematica package `BVPh` (version 1.0) is developed by the author in the frame of the HAM for nonlinear boundary-value problems. A dozen of examples are used to illustrate its validity for highly nonlinear ODEs with singularity, multiple solutions and multipoint boundary conditions in either a finite or an infinite interval, and even for some types of nonlinear PDEs. As an open resource, the `BVPh` 1.0 is given in this book with a simple users guide and is free available online.

In Part III, we illustrate that the HAM can be used to solve some complicated highly nonlinear PDEs so as to enrich and deepen our understandings about these interesting nonlinear problems. For example, By means of the HAM, an explicit analytic approximation of the optimal exercise boundary of American put option was gained, which is often valid for a couple of decades prior to expiry, whereas the asymptotic and perturbation formulas are valid only for a couple of days or weeks in general. A Mathematica code based on such kind of explicit formula is given in this book for businessmen to gain accurate results in a few seconds. In addition, by means of the HAM, the wave-resonance criterion of arbitrary number of traveling gravity waves was found, for the first time, which logically contains the famous Phillips' criterion for four waves with small amplitude.

All of these show the originality, validity and generality of the HAM for highly nonlinear problems in science, finance and engineering.

All Mathematica codes and their input data files are given in the appendixes of this book and available (Accessed 25 Nov 2011, will update in the future) either at

http://numericaltank.sjtu.edu.cn/HAM.htm

or at

http://numericaltank.sjtu.edu.cn/BVPh.htm

This book is suitable for researchers and postgraduates in applied mathematics, physics, finance and engineering, who are interested in highly nonlinear ODEs and PDEs.

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March 2011 *Shijun Liao*