

Contents

Part I Basic Ideas and Theorems

1	Introduction	3
1.1	Motivation and purpose	3
1.2	Characteristic of homotopy analysis method	5
1.3	Outline	6
	References	8
2	Basic Ideas of the Homotopy Analysis Method	15
2.1	Concept of homotopy	15
2.2	Example 2.1: generalized Newtonian iteration formula	19
2.3	Example 2.2: nonlinear oscillation	26
2.3.1	Analysis of the solution characteristic	26
2.3.2	Mathematical formulations	31
2.3.3	Convergence of homotopy-series solution	38
2.3.4	Essence of the convergence-control parameter c_0	48
2.3.5	Convergence acceleration by homotopy-Padé technique ...	56
2.3.6	Convergence acceleration by optimal initial approximation	59
2.3.7	Convergence acceleration by iteration	63
2.3.8	Flexibility on the choice of auxiliary linear operator	69
2.4	Concluding remarks and discussions	75
Appendix 2.1	Derivation of δ_n in (2.57)	79
Appendix 2.2	Derivation of (2.55) by the 2nd approach	80
Appendix 2.3	Proof of Theorem 2.3	82
Appendix 2.4	Mathematica code (without iteration) for Example 2.2 ...	83
Appendix 2.5	Mathematica code (with iteration) for Example 2.2	87
	Problems	92
	References	92

3	Optimal Homotopy Analysis Method	95
3.1	Introduction	95
3.2	An illustrative description	101
3.2.1	Basic ideas	101
3.2.2	Different types of optimal methods	104
3.3	Systematic description	117
3.4	Concluding remarks and discussions	121
	Appendix 3.1 Mathematica code for Blasius flow	122
	Problems	126
	References	127
4	Systematic Descriptions and Related Theorems	131
4.1	Brief frame of the homotopy analysis method	131
4.2	Properties of homotopy-derivative	133
4.3	Deformation equations	148
4.3.1	A brief history	148
4.3.2	High-order deformation equations	153
4.3.3	Examples	165
4.4	Convergence theorems	168
4.5	Solution expression	173
4.5.1	Choice of initial approximation	175
4.5.2	Choice of auxiliary linear operator	176
4.6	Convergence control and acceleration	179
4.6.1	Optimal convergence-control parameter	180
4.6.2	Optimal initial approximation	181
4.6.3	Homotopy-iteration technique	181
4.6.4	Homotopy-Padé technique	182
4.7	Discussions and open questions	183
	References	185
5	Relationship to Euler Transform	189
5.1	Introduction	189
5.2	Generalized Taylor series	190
5.3	Homotopy transform	210
5.4	Relation between homotopy analysis method and Euler transform	215
5.5	Concluding remarks	219
	References	220
6	Some Methods Based on the HAM	223
6.1	A brief history of the homotopy analysis method	223
6.2	Homotopy perturbation method	225
6.3	Optimal homotopy asymptotic method	228
6.4	Spectral homotopy analysis method	230
6.5	Generalized boundary element method	230
6.6	Generalized scaled boundary finite element method	231

6.7	Predictor homotopy analysis method	232
	References	233

Part II Mathematica Package BVPh and Its Applications

7	Mathematica Package BVPh	239
7.1	Introduction	239
7.1.1	Scope	242
7.1.2	Brief mathematical formulas	243
7.1.3	Choice of base function and initial guess	247
7.1.4	Choice of the auxiliary linear operator	250
7.1.5	Choice of the auxiliary function	252
7.1.6	Choice of the convergence-control parameter c_0	253
7.2	Approximation and iteration of solutions	254
7.2.1	Polynomials	255
7.2.2	Trigonometric functions	256
7.2.3	Hybrid-base functions	257
7.3	A simple users guide of the BVPh 1.0	260
7.3.1	Key modules	260
7.3.2	Control parameters	261
7.3.3	Input	263
7.3.4	Output	264
7.3.5	Global variables	264
	Appendix 7.1 Mathematica package BVPh (version 1.0)	265
	References	278
8	Nonlinear Boundary-value Problems with Multiple Solutions	285
8.1	Introduction	285
8.2	Brief mathematical formulas	286
8.3	Examples	289
8.3.1	Nonlinear diffusion-reaction model	289
8.3.2	A three-point nonlinear boundary-value problem	296
8.3.3	Channel flows with multiple solutions	301
8.4	Concluding remarks	306
	Appendix 8.1 Input data of BVPh for Example 8.3.1	307
	Appendix 8.2 Input data of BVPh for Example 8.3.2	309
	Appendix 8.3 Input data of BVPh for Example 8.3.3	310
	Problems	312
	References	313
9	Nonlinear Eigenvalue Equations with Varying Coefficients	315
9.1	Introduction	315
9.2	Brief mathematical formulas	317
9.3	Examples	322
9.3.1	Non-uniform beam acted by axial load	322
9.3.2	Gelfand equation	333

9.3.3	Equation with singularity and varying coefficient	337
9.3.4	Multipoint boundary-value problem with multiple solutions	342
9.3.5	Orr-Sommerfeld stability equation with complex coefficient	346
9.4	Concluding remarks	350
Appendix 9.1	Input data of BVPh for Example 9.3.1	351
Appendix 9.2	Input data of BVPh for Example 9.3.2	353
Appendix 9.3	Input data of BVPh for Example 9.3.3	354
Appendix 9.4	Input data of BVPh for Example 9.3.4	355
Appendix 9.5	Input data of BVPh for Example 9.3.5	357
	Problems	358
	References	359
10	A Boundary-layer Flow with an Infinite Number of Solutions	363
10.1	Introduction	363
10.2	Exponentially decaying solutions	365
10.3	Algebraically decaying solutions	369
10.4	Concluding remarks	376
Appendix 10.1	Input data of BVPh for exponentially decaying solution	377
Appendix 10.2	Input data of BVPh for algebraically decaying solution	378
	References	380
11	Non-similarity Boundary-layer Flows	383
11.1	Introduction	383
11.2	Brief mathematical formulas	387
11.3	Homotopy-series solution	392
11.4	Concluding remarks	396
Appendix 11.1	Input data of BVPh	397
	References	399
12	Unsteady Boundary-layer Flows	403
12.1	Introduction	403
12.2	Perturbation approximation	406
12.3	Homotopy-series solution	408
12.3.1	Brief mathematical formulas	408
12.3.2	Homotopy-approximation	412
12.4	Concluding remarks	417
Appendix 12.1	Input data of BVPh	418
	References	420

Part III Applications in Nonlinear Partial Differential Equations

13 Applications in Finance: American Put Options	425
13.1 Mathematical modeling	425
13.2 Brief mathematical formulas	428
13.3 Validity of the explicit homotopy-approximations	436
13.4 A practical code for businessmen	443
13.5 Concluding remarks	444
Appendix 13.1 Detailed derivation of $f_n(\tau)$ and $g_n(\tau)$	446
Appendix 13.2 Mathematica code for American put option	448
Appendix 13.3 Mathematica code APOh for businessmen	454
References	457
14 Two and Three Dimensional Gelfand Equation	461
14.1 Introduction	461
14.2 Homotopy-approximations of 2D Gelfand equation	462
14.2.1 Brief mathematical formulas	462
14.2.2 Homotopy-approximations	468
14.3 Homotopy-approximations of 3D Gelfand equation	474
14.4 Concluding remarks	480
Appendix 14.1 Mathematica code of 2D Gelfand equation	481
Appendix 14.2 Mathematica code of 3D Gelfand equation	485
References	489
15 Interaction of Nonlinear Water Wave and Nonuniform Currents	493
15.1 Introduction	493
15.2 Mathematical modeling	494
15.2.1 Original boundary-value equation	494
15.2.2 Dubreil-Jacotin transformation	496
15.3 Brief mathematical formulas	497
15.3.1 Solution expression	497
15.3.2 Zeroth-order deformation equation	498
15.3.3 High-order deformation equation	500
15.3.4 Successive solution procedure	502
15.4 Homotopy approximations	504
15.5 Concluding remarks	515
Appendix 15.1 Mathematica code of wave-current interaction	516
References	521
16 Resonance of Arbitrary Number of Periodic Traveling Water Waves	523
16.1 Introduction	523
16.2 Resonance criterion of two small-amplitude primary waves	525
16.2.1 Brief Mathematical formulas	525
16.2.2 Non-resonant waves	533
16.2.3 Resonant waves	538

16.3 Resonance criterion of arbitrary number of primary waves	547
16.3.1 Resonance criterion of small-amplitude waves	547
16.3.2 Resonance criterion of large-amplitude waves	550
16.4 Concluding remark and discussions	553
Appendix 16.1 Detailed derivation of high-order equation	555
References	561
Index	563