6 Some other approaches based on the HAM

6.1 The spectral HAM

In 2010, Motsa et al. [35, 36] suggested the so-called "spectral homotopy analysis method" (SHAM) by using the Chebyshev pseudo-spectral method to solve the linear high-order deformation equations. Since the SHAM combines the HAM with the numerical techniques, it provides us larger freedom to choose auxiliary linear operators. Thus, one can choose more complicated auxiliary linear operators in the frame of the SHAM.

In theory, any a continuous function in a bounded interval can be best approximated by Chebyshev polynomial. So, the SHAM provides larger freedom to choose the auxiliary linear operator $\mathcal{L}$ and initial guess. The basic idea of the SHAM might be expanded to solve nonlinear partial differential equations. Besides, it is easy to employ the optimal convergence-control parameter in the frame of the SHAM. Thus, the SHAM has great potential to solve more complicated nonlinear problems in science and engineering, although further modifications in theory and more applications are needed.

Chebyshev polynomial is a kind of special function. There are many other special functions such as Hermite polynomial, Legendre polynomial, Airy function, Bessel function, Riemann zeta function, hypergeometric functions and so on. Since the HAM provides us extremely large freedom to choose the auxiliary linear operator $\mathcal{L}$ and the initial guess, it should be possible to develop a "generalized spectral HAM" which can use a proper special function for a given nonlinear problem.

6.2 The predictor HAM

Abbasbandy et al. [1–3] proposed the so-called "the predictor homotopy analysis method" (PHAM) to predict the multiplicity of solutions of nonlinear equations. Using the PHAM, they obtained multiple solutions of some nonlinear differential equations by means of different values of the convergence-control parameter c_0 with the **same** auxiliary linear operator $\mathcal{L}$ and even the **same** initial guess. As pointed out by Abbasbandy et al. [2], this trait makes HAM to be different from the other analytical techniques which are used to approach one solution but possibly lose the others.

For details, please refer to Abbasbandy et al. [1–3].