

10.3 Algebraically decaying boundary-layer flow

Consider here a two-dimensional boundary-layer viscous flow in the region $x > 0$ and $y > 0$, where (x, y) denotes a Cartesian coordinate system and the flow results solely from the movement of an impermeable flat plate at $y = 0$ in its plane. Let $U_w(x) = a(x+b)^\kappa$ denote the speed of the flat plate, where $a > 0$, $b > 0$ are given constants. Assume that the boundary-layer equations are appropriate so that such kind of flow is described by the partial differential equations (PDEs)

$$u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \nu \frac{\partial^2 u}{\partial y^2}, \quad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

subject to the boundary conditions

$$\begin{aligned} u &= U_w(x), \quad v = 0, \quad \text{at } y = 0, \\ u &= 0, \quad \text{at } y \rightarrow +\infty, \end{aligned}$$

where ν is the kinematic viscosity and u, v are the velocity components in the directions of increasing x, y , respectively.

Let ψ denote the stream function. Using the similarity transformation

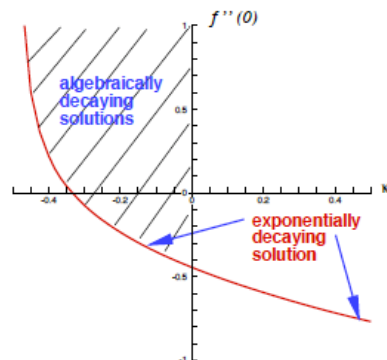
$$\psi = \sqrt{av}(x+b)^{(\kappa+1)/2} f(\eta), \quad \eta = \sqrt{\frac{a}{\nu}} y(x+b)^{(\kappa-1)/2},$$

the original PDEs become the following nonlinear ODE

$$f''' + \frac{(1+\kappa)}{2} f f'' - \kappa f'^2 = 0, \quad f(0) = 0, f'(0) = 1, f'(+\infty) = 0. \quad (10.3)$$

Note that this problem is defined in an **infinite** interval. For given $-1/2 < \kappa < 0$, this nonlinear ODE has an infinite number of **algebraically decaying** solutions, which can be found out by means of the **BVPh 1.0**, as shown below.

Fig. 10.4 $f''(0)$ versus κ for the exponentially and algebraically decaying solutions of (10.3). Solid line: exponentially decaying solutions; Dashed area: algebraically decaying solutions.



Note that these algebraically decaying solutions were found out **first** by Liao and Magyari **using the HAM** in 2006, which had been neglected by other analytic and numerical methods.