11. Non-similarity boundary-layer flow

Consider here a non-similarity boundary-layer flow of Newtonian fluid over a stretching flat sheet. Two forces in opposite directions with same magnitude are added along the sheet. Due to the symmetry of flows, we can only consider the flows in the upper quarter plane $x \ge 0$ and $y \ge 0$. Let $U_w(x)$ denote the stretching velocity of the sheet. The problem is governed by

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=\nu\frac{\partial^2 u}{\partial y^2}, \qquad \frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0,$$

subject to the boundary conditions

$$u=U_w(x), \quad v=0, \quad \text{at} \quad y=0, \qquad u=0, \qquad \text{at} \quad y\to+\infty,$$
$$u=0, \quad \frac{\partial v}{\partial x}=0, \quad \text{at} \ \ x=0,$$

where ν is the kinematic viscosity and u, v are the velocity components in the directions of increasing x, y, respectively.

Let ψ denote the stream function. When $U_w(x)=x/(1+x)$, there exist no similarity solution. In this case, using the transformation

$$\psi=\sqrt{\nu(1+x)}\,f(\eta,\xi), \qquad \eta=\frac{y}{\sqrt{\nu(1+x)}}, \qquad \xi=\frac{x}{1+x},$$

the original PDEs become the following nonlinear PDE

$$\frac{\partial^3 f}{\partial \eta^3}+\frac{1}{2}f\frac{\partial^2 f}{\partial \eta^2}+(1-\xi)\left(\frac{\partial f}{\partial \xi}\frac{\partial^2 f}{\partial \eta^2}-\frac{\partial f}{\partial \eta}\frac{\partial^2 f}{\partial \xi\partial \eta}\right)=0, \qquad (11.17)$$

subject to the boundary conditions

$$f(0,\xi)=0, \quad f_\eta(0,\xi)=\xi, \quad f_\eta(+\infty,\xi)=0. \qquad (11.18)$$

Even such kind of nonlinear PDE in an infinite interval can be solved by means of the **BVPh 1.0**, as shown below.

Fig. 11.5 The velocity profile $u \sim y/\sqrt{\nu}$ of the non-similarity flow at different x. Solid line: $x=1/4$; Dashed line: $x=1/2$; Dash-dotted line: $x=1$; Dash-dot-dotted line: $x=5$; Long-dashed line: $x=10$.