Example 8.3.3 Channel flows with multiple solutions

Consider a steady laminar flow of a viscous incompressible fluid in a two-dimensional porous channel with uniform injection or suction, described by a nonlinear boundary-value differential equation

$$u''''+\alpha\left(zu'''+3u''\right)+R\left(uu'''-u'u''\right)=0, \qquad u(0)=0,u''(0)=0,u(1)=1,u'(1)=0,$$

where the prime denotes the differentiation with respect to z, R is the cross-flow Reynolds number, α is a physical constant related to the wall expansion ratio, respectively, and $u(z)$ is related to the stream function

$$\psi(x,y)=\frac{\upsilon x}{Rd}u(z), \qquad z=\frac{y}{d}.$$

This equation may have three solutions as shown below, which can be found out by means of the **BVPh 1.0**.