9.3.5 Orr-Sommerfeld stability equation with complex coefficient

Consider the famous Orr-Sommerfeld stability equation for the stability of plane Poiseuille flow

$$\left(D^2-\alpha^2\right)^2 u - i(\alpha R)\left[\left(U_0-\lambda\right)\left(D^2-\alpha^2\right)-D^2U_0\right]u = 0\,, \tag{9.62}$$

subject to the boundary conditions

$$u'(0)=u'''(0)=0, \quad u(1)=0, u'(1)=0, \tag{9.63}$$

where the prime denotes the differentiation with respect to z, $i=\sqrt{-1}$ is an imaginary number, the operator D is defined by $Du = u''$, R denotes the Reynolds number, λ is the complex eigenvalue, $U_0 = 1 - z^2$ is the exact solution of the plane Poiseuille flow, respectively.

This eigenvalue equation contains a complex coefficient. It can be solved by means of the **BVPh 1.0**, as shown below.

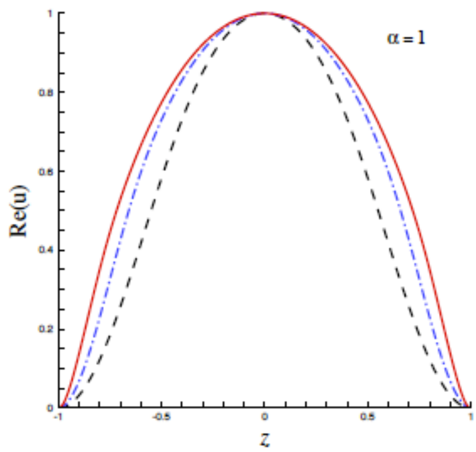

Fig. 9.23 The real part of the eigenfunctions of the Orr-Sommerfeld stability equation (9.62) when $\alpha = 1$. Dashed line: $R = 100$; Dash-dotted line: $R = 1000$; Solid line: $R = 5814.83$.

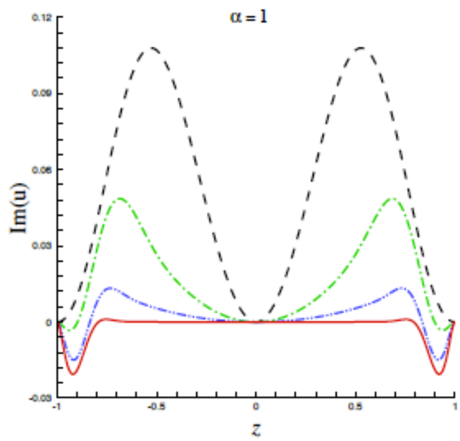

Fig. 9.24 The imaginary part of the eigenfunctions of the Orr-Sommerfeld stability equation (9.62) when $\alpha = 1$. Dashed line: $R = 100$; Dash-dotted line: $R = 1000$; Dash-dot-dotted line: $R = 3000$; Solid line: $R = 5814.83$.