Preface

In general it is difficult to obtain analytic approximations of nonlinear problems with strong nonlinearity. Traditionally, solution expressions of a nonlinear problem are mainly determined by the type of nonlinear equations and the employed analytic techniques, and the convergence regions of solution series are strongly dependent of physical parameters. It is well known that analytic approximations of nonlinear problems often break down as nonlinearity becomes strong and perturbation approximations are valid only for nonlinear problems with weak nonlinearity.

In this book we introduce an analytic method for nonlinear problems in general, namely the homotopy analysis method. We show that, even if a nonlinear problem has a unique solution, there may exist an *infinite* number of different solution expressions whose convergence region and rate are dependent on an auxiliary parameter. Unlike all previous analytic techniques, the homotopy analysis method provides us with a simple way to *control* and *adjust* the convergence region and rate of solution series of nonlinear problems. Thus, this method is valid for nonlinear problems with strong nonlinearity. Moreover, unlike all previous analytic techniques, the homotopy analysis method provides great freedom to use different base functions to express solutions of a nonlinear problem so that one can approximate a nonlinear problem more efficiently by means of better base functions. Furthermore, the homotopy analysis method logically contains some previous techniques such as Adomian's decomposition method, Lyapunov's artificial small parameter method, and the δ-expansion method. Thus, it can be regarded as a unified or generalized theory of these previous methods.

The book consists of two parts. Part I (Chapter 1 to Chapter 5) deals with the basic ideas of the homotopy analysis method. In Chapter 2, the homotopy analysis method is introduced by means of a rather simple nonlinear problem. The reader is strongly advised to read this chapter first. In Chapter 3, a systematic description is given and a convergence theorem is described for general cases. In Chapter 4 we show that Lyapunov's artificial small parameter method, the δ-expansion method, and Adomian's decomposition method are simply special cases of the homotopy analysis method. In Chapter 5 the advantages and limitations of the homotopy analysis method are briefly discussed and some open questions are pointed out. In Part II (Chapter 6 to Chapter 18), the homotopy analysis method is applied to solve some nonlinear problems, such as simple bifurcations of a nonlinear boundary-value problem (Chapter 6), multiple solutions of a nonlinear boundary-value problem (Chapter 7), eigenvalue and eigenfunction of a nonlinear boundary-value problem (Chapter 8), the Thomas-Fermi atom model (Chapter 9), Volterra's population model (Chapter 10), free oscillations of conservative systems with odd nonlinearity (Chapter 11), free oscillations of conservative systems with quadratic nonlinearity (Chapter 12), limit cycle in a multidimensional system (Chapter 13), Blasius' viscous flow (Chapter 14), boundary-layer flows with exponential property (Chapter 15), boundary-layer flows with algebraic property (Chapter 16), Von Kármán swirling viscous flow (Chapter 17), and nonlinear progressive waves in deep water (Chapter 18). In Part II, only Chapters 14, 15, and 18 are adapted from published articles of the author.

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