10.2 Unique exponentially decaying boundary-layer flow

Consider here a two-dimensional boundary-layer viscous flow in the region $x > 0$ and $y > 0$, where (x,y) denotes a Cartesian coordinate system and the flow results solely from the movement of an impermeable flat plate at $y = 0$ in its plane. Let $U_w(x) = a(x+b)^\kappa$ denote the speed of the flat plate, where $a > 0$, $b > 0$ are given constants. Assume that the boundary-layer equations are appropriate so that such kind of flow is described by the partial differential equations (PDEs)

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2}, \qquad \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 ,$$

subject to the boundary conditions

$$\begin{aligned} &u = U_x(x), \quad v = 0, \quad \text{at} \quad y = 0, \\ &u = 0, \qquad\quad \text{at} \quad y \to +\infty, \end{aligned}$$

where ν is the kinematic viscosity and u, v are the velocity components in the directions of increasing x, y, respectively.

Let ψ denote the stream function. Using the similarity transformation

$$\psi = \sqrt{a\nu}\,(x+b)^{(\kappa+1)/2}\, f(\eta), \qquad \eta = \sqrt{\frac{a}{\nu}}\, y(x+b)^{(\kappa-1)/2} ,$$

the original PDEs become the following nonlinear ODE

$$f''' + \frac{(1+\kappa)}{2} f f'' - \kappa f'^2 = 0, \qquad f(0) = 0, f'(0) = 1, f'(+\infty) = 0 . \tag{10.3}$$

Note that this problem is defined in an infinite interval. For given κ, this nonlinear ODE has an exponentially decaying solution, which can be found out by means of the **BVPh 1.0**, as shown below.

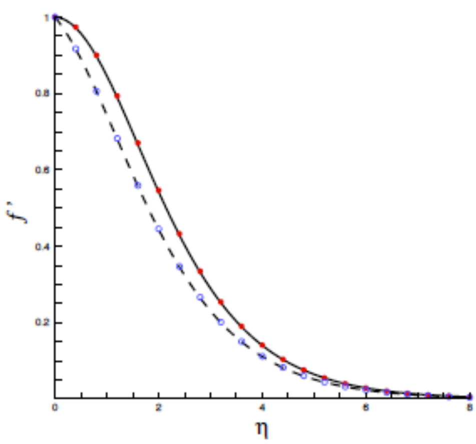

Fig. 10.1 The exponentially decaying solutions $f'(\eta)$ of (10.3). Filled circle: the 10th-order homotopy approximation when $\kappa = -1/3$ by means of $c_0 = -5/4$ and $\sigma = 3$; Solid line: the exact solution when $\kappa = -1/3$; Open circle: the 10th-order homotopy approximation when $\kappa = -1/4$ by means of $c_0 = -5/4$ and $\sigma = 11/4$; Dashed line: the 30th-order homotopy approximation when $\kappa = -1/4$.