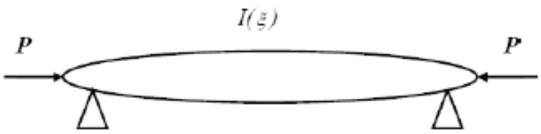
9.3.1 Non-uniform beam acted by axial load

Fig. 9.1 Beam acted by an axial load P with variable moment of inertia $I(z)$



Consider a non-uniform beam with arbitrary cross-section on two supports under an axial load P, as shown in Fig. 9.1, where P is positive for compressive force and negative for tensile force. Let l and θ denote the length and slope of the beam, and u its deflection, respectively. Let s denote the arc-coordinate of the natural axis which passes through the centroid of each cross-section of the beam in its straight or unbuckled state, $I(s)$ the smallest moment of inertia of the cross-section about a line in its plane through the centroid, E the Young's modulus of the material, respectively. Assume that all of the principle axes of inertia are parallel so that the beam is not twisted. Mathematically, the problem is governed by

$$[\mu(z)\theta'(z)]' + \lambda \sin\theta = 0, \qquad \theta'(0) = \theta'(\pi) = 0 ,$$

where the prime denotes the differentiation with respect to z, $\mu(z) > 0$ is an arbitrary continuous function, and

$$\lambda = \frac{P}{EI_0}\left(\frac{l}{\pi}\right)^2$$

is regarded as a unknown eigenvalue.

For given $\gamma = \theta(0) \neq 0$, the above nonlinear eigenvalue problem has two solutions as shown below, which can be found out by means of the **BVPh 1.0**

Fig. 9.5 Displacement given by two different eigenfunctions when $\mu = 1, \gamma = 1$ and $\kappa = 1$. Solid line: displacement corresponding to $\lambda = 1.137069$ given by $\theta_0(z) = \cos(z)$; dashed line: displacement corresponding to $\lambda = -1.951368$ given by $\theta_0(z) = 3 - 2\cos(z)$.

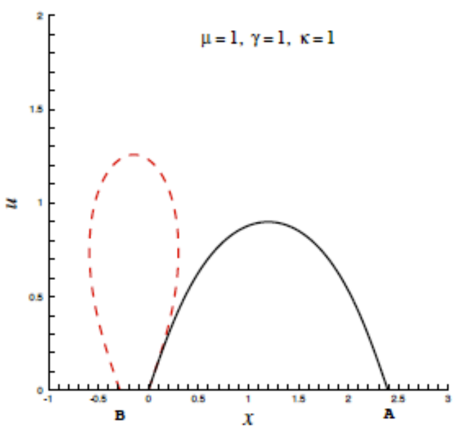


Fig. 9.8 Displacement given by two different types of eigenfunctions when $\mu = 1$ and $\kappa = 1$. Solid line: displacement corresponding to the 1st-type eigenfunction with the eigenvalue $\lambda^* = 3.202901$ given by $\gamma^* = \pi - 1/2$; dashed line: displacement corresponding to the 2nd-type eigenfunction with the eigenvalue $\bar{\lambda} = -3.202901$ given by $\bar{\gamma} = 1/2$.

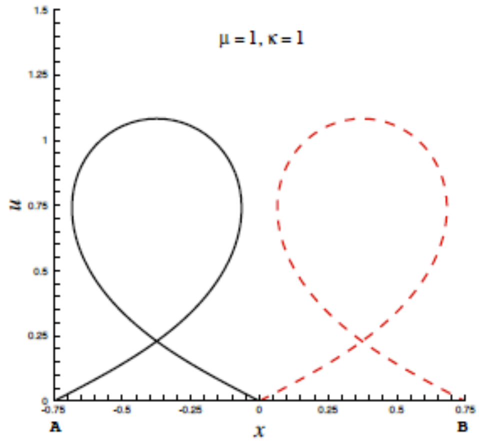
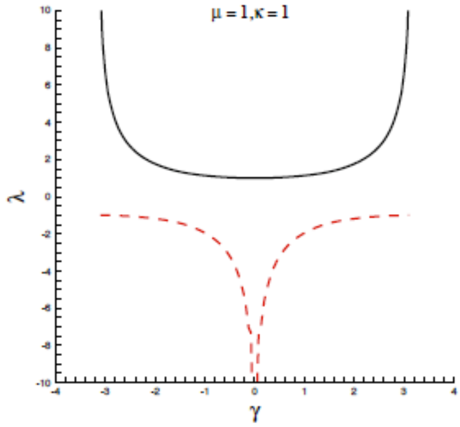


Fig. 9.9 Eigenvalue versus $\gamma = \theta(0)$ when $\mu = 1$ and $\kappa = 1$. Solid line: positive eigenvalue corresponding to the 1st-type eigenfunction; dashed line: negative eigenvalue corresponding to the 2nd-type eigenfunction.



The multiple solutions can be found out by means of the **BVPh 1.0** even for non-uniform beam with rather complicated cross-section such as

$$\mu(z) = 1 + \frac{\cos(4z)}{2\sqrt{1 + \exp(z^2) + \sin(z^2)}}$$