9.3.3 An eigenvalue equation with singularity and varying coefficients

Consider a nonlinear eigenvalue equation with varying coefficients defined in a finite interval $0 < z < \pi$:

$$\sqrt{1+z^2}\; u'' + \frac{\cos(\pi z)\, u'}{z} + \lambda\left[\frac{\exp(u)}{1+z^2} + (1+z)\sin u\right] = \sin(z^2 + e^{-z}), \qquad (9.44)$$

subject to the boundary conditions

$$u'(0) = 0, \quad u(\pi) - u'(\pi) = \frac{3}{5}, \qquad (9.45)$$

where the prime denotes the differentiation with respect to z, $u(z)$ and λ are the unknown eigenfunction and eigenvalue, respectively.

This eigenvalue equation contains rather complicated varying coefficient and the highly nonlinear terms $\exp(u)$ and $\sin(u)$. In addition, it has a singularity at $z = 0$ due to the term $u'(z)/z$. Such kind of singularity leads to difficulty to numerical techniques such as the shooting method used by BVP4c. Thus, this equation is rather complicated.

Write $A = u(0)$. This complicated nonlinear eigenvalue equation can be solved by means of the BVPh 1.0, as shown below.

Fig. 9.16 Eigenfunction of (9.44) and (9.45) by means of $\kappa = 1$. Solid line: 10th-iteration approximation when $A = 1/2$ by means of $c_0 = -2/5$; Dashed line: 10th-iteration approximation when $A = 0$ by means of $c_0 = -2/5$; Dash-dotted line: 15th-iteration approximation when $A = -1/2$ by means of $c_0 = -1/5$. Symbols: the 3rd-iteration approximations ($A = 1/2$ and $A = 0$) or the 8th-iteration approximation ($A = -1/2$).

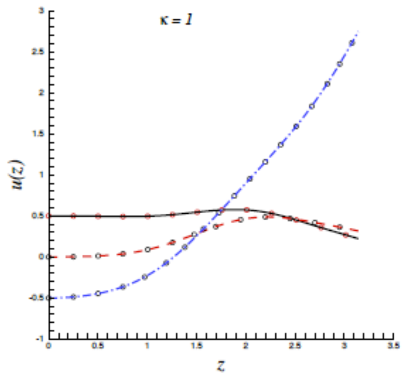

Fig. 9.17 Eigenvalue of (9.44) and (9.45) versus A by means of $\kappa = 1$.