9.3.4 Multipoint boundary value problem with multiple solutions

Consider here a 4th-order nonlinear eigenvalue problem with multipoint boundary condition

$$u'''' = \beta z(1+u^2), \quad u(0) = u'(1) = u''(1) = 0, \; u''(0) = u''(\alpha), \tag{9.52}$$

where $\alpha \in (0,1)$ and β are given constants. Writing $u(z) = \lambda\theta(z)$ with the definition $\lambda = u(1)$, the original equation (9.52) becomes

$$\theta'''' = \beta z(1+\lambda^2\theta^2), \quad \theta(0) = \theta'(1) = \theta''(1) = 0, \; \theta''(0) = \theta''(\alpha), \tag{9.53}$$

with one additional boundary condition

$$\theta(1) = 1. \tag{9.54}$$

The two solutions can be found out by means of the **BVPh 1.0**, as shown below.

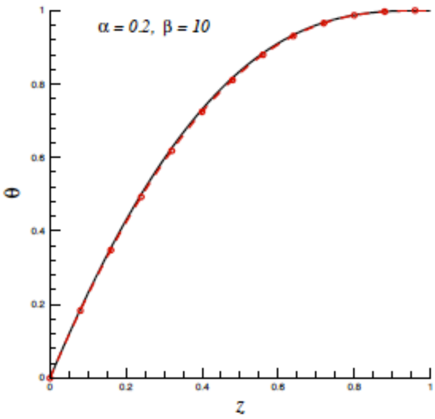

Fig. 9.21 Comparison of two eigenfunctions of (9.53) and (9.54) when $\alpha = 1/5$ and $\beta = 10$ by means of different initial guesses λ_0 of the eigenvalue. Solid line: the 1st eigenfunction $\theta(z)$ given by $\lambda_0 = 0.63313$ and $c_0 = -3/2$; Dashed line with open circles: the 2nd eigenfunction $\theta(z)$ given by $\lambda_0 = 2.14591$ and $c_0 = -1/2$.

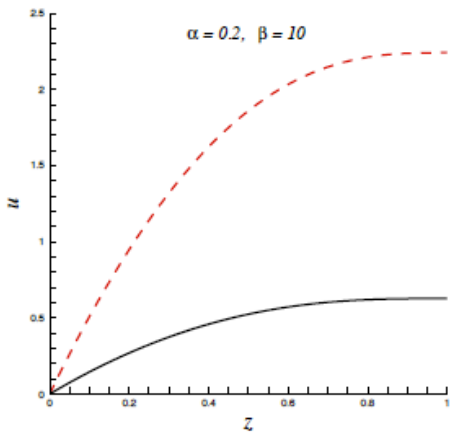

Fig. 9.22 Two solutions of the original equation (9.52) when $\alpha = 1/5$ and $\beta = 10$ Solid line: 1st solution $u(z)$ given by $\lambda_0 = 0.63313$ and $c_0 = -3/2$; Dashed line: 2nd solution $u(z)$ given by $\lambda_0 = 2.14591$ and $c_0 = -1/2$.